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# Influence of solar chaotic magnetic fields on neutrino oscillations

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## Abstract

We consider the effect of a random magnetic field in the convective zone of the Sun on resonant neutrino spin-flavour oscillations. The expected signals in the different experiments (SK, GALLEX-SAGE, Homestake) are obtained as a function of the level of noise, regular magnetic field and neutrino mixing parameters. Previous results obtained for small mixing and ad-hoc regular magnetic profiles are reobtained. We find that MSW regions are stable up to very large levels of noise ( $P=0.7-0.8$ ) and they are acceptable from the point of view of antineutrino production. For strong noise any parameter region ( $\Delta m^2, \sin^2 2\theta$ ) is excluded: this model of noisy magnetic field is not compatible with particle physics solutions to the SNP.

## 1 Introduction

A neutrino transition magnetic moment can account, both for the observed deficiency of the solar neutrino flux and the time variations of the signal. The overall deficit is caused by the suppression of the neutrino energy spectrum. The time dependence may be caused by time variations of the convective magnetic field of the Sun [1]. In transverse magnetic fields, neutrinos with transition magnetic moments will experience spin and flavor rotation simultaneously (resonant spin-flavor precession, RSFP). The observation of electron antineutrinos from the Sun would lead to the conclusion that the neutrinos are Majorana particles. There are however stringent bounds on the presence of solar electron antineutrinos coming from solar  $^8\text{B}$  neutrinos [2, 3].

Magnetic fields measured on the surface of the Sun are weaker than those ones within the interior of the convective zone. The mean field value over the solar disc is of the order of 1 G and in the solar spots magnetic field strength reaches 1 kG. In magnetic hydrodynamics, MHD, one can explain such fields in a self-consistent way if these fields are generated by the existence of a dynamo effect at the bottom of the convective zone. In this region the strength of small scale regular magnetic fields could reach a value of 100 kG. These fields propagate through the convective zone and photosphere decreasing

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in strength while increasing in the scale giving traces in form of loops in bipolar active regions (solar spots).

Large-scale toroidal magnetic field created by a dynamo mechanism in convective zone typically have strengths even less than small-scale r.m.s fields near the bottom of convective zone. This is the main reason why one should consider neutrino propagation in the random magnetic field of the Sun. The ratio of the r.m.s. random field and the regular (toroidal) field may be about  $\sim 20$ -50, therefore the problem of RSFP neutrino propagation in noisy magnetic field seems to be important. Estimations for the ratio of rms fields to regular field are necessarily very rough. In textbooks [4, 5, 6, 7] we find the conservative ratio  $\eta \equiv \langle \tilde{B}^2 \rangle / B_0^2 \sim 1$ . In more elaborate models the ratio of magnetic energy densities is given by the magnetic Reynolds number,  $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m^\sigma$ , which may be much bigger than unity for plasma with large conductivity. Here  $\sigma > 0$  is a topology index [4, 5, 8, 9].

The effect of random magnetic fields in RSFP solutions to the solar neutrino problem, SNP, and antineutrino production has been explored previously for simplified models [10, 11]. In this work we will deal with the complete problem, we will present calculations of neutrino spin flavor conversions in presence of matter and magnetic field. The magnetic field will have two well differentiated components. The first component will be a theoretically motivated solar magnetic field profile: solution to the static magnetic hydrodynamic equations. As a second component, we include a layer of magnetic noise generated at the bottom of the convective zone, we have justified that the level of noise in this region can certainly be very high.

## 2 The Master equation

We consider conversions  $\nu_{eL,R} \rightarrow \nu_{aL,R}$ ,  $a = \mu$  or  $\tau$ , (for definiteness we will refer to  $\mu$  for the rest of this work) for two neutrino flavors obeying the following master evolution equation

$$i\partial_t \begin{pmatrix} \nu_{eL} \\ \bar{\nu}_{eR} \\ \nu_{\mu L} \\ \bar{\nu}_{\mu R} \end{pmatrix} = \begin{pmatrix} V_e - c_2\delta & 0 & s_2\delta & \mu B_\perp^+(t) \\ 0 & -V_e - c_2\delta & -\mu B_\perp^-(t) & s_2\delta \\ s_2\delta & -\mu B_\perp^+(t) & V_\mu + c_2\delta & 0 \\ \mu B_\perp^-(t) & s_2\delta & 0 & -V_\mu + c_2\delta \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ \bar{\nu}_{eR} \\ \nu_{\mu L} \\ \bar{\nu}_{\mu R} \end{pmatrix}, \quad (1)$$

where  $c_2 = \cos 2\theta$ ,  $s_2 = \sin(2\theta)$  and  $\delta = \Delta m^2/4E$  are the neutrino mixing parameters;  $\mu \equiv \mu_{12}$  is the neutrino active-active transition magnetic moment,

$$B_\perp^\pm(t) = B_{0\perp}^\pm(t) + \tilde{B}_\perp^\pm(t) \quad (2)$$

is the magnetic field component which is perpendicular to the neutrino trajectory in the Sun. The quantities  $V_{e,\mu}(t)$  are the neutrino vector potentials for  $\nu_{eL}$  and  $\nu_{\mu L}$  in the Sun given by the abundances of the electron and neutron components and by the SSM density profile [12].

The transverse magnetic field  $B_\perp(t)$  appearing in Eq. (2) is given by  $B_\perp^\pm(t) \equiv B_x(t) \pm iB_y(t) \equiv |B_\perp(t)|\text{Exp}(\pm i\Phi(t))$ . The nature and magnitude for the regular ( $B_0$ ) and chaotic

parts ( $\tilde{B}$ ) of the magnetic field will be the subject of the next section. In our calculations we will consider always the product  $\mu B$ . Expected values of  $B \approx 1 - 100$  kG in the Sun convective zone and  $\mu = 10^{-11} \mu_B$  would give an expected range for the product  $\mu B \approx 10^{-8} - 10^{-6} \mu_B G \approx 5.6 \cdot 10^{-17} - 10^{-15}$  eV or in the practical units which will be used throughout this work  $\mu B \approx 0.1 - 10.0 \mu_{11} B_4$ .

### 3 Solar magnetic fields

#### 3.1 Random magnetic fields

The r.m.s. random component  $\sqrt{\langle \tilde{B}^2(t) \rangle}$  can be comparable in magnitude with the regular one,  $B_0(t)$ , and maybe even much stronger than  $B_0$ , if a large magnetic Reynolds number  $R_m$  leads to the effective dynamo enhancement of small-scale (random) magnetic fields.

Let us give some simple estimates of the magnetic Reynolds number  $R_m = lv/\nu_m$  in the convective zone for fully ionized hydrogen plasma ( $T \gg I_H \sim 13.6$  eV  $\sim 10^5$  K). Here  $l \sim 10^8$  cm is the size of eddy (of the order of magnitude of a granule size) with the turbulent velocity inside of it  $v \sim v_A \sim 10^5$  cm/s where  $v_A = B_0/\sqrt{4\pi\rho}$  is the Alfven velocity for MHD plasma,  $B_0$  is a large-scale field in the zone and  $\rho$  is the matter density (in g/cm<sup>3</sup>).

The magnetic diffusion coefficient,  $\nu_m = c^2/4\pi\sigma_{cond}$ , appears in the diffusion term of the Faraday equation:

$$\frac{\partial \vec{B}(t)}{\partial t} = \text{rot}[\vec{v} \times \vec{B}(t)] + \nu_m \Delta \vec{B}. \quad (3)$$

Here  $c$  is the light speed. The conductivity of the hydrogen plasma is  $\sigma_{cond} = \omega_{pl}^2/4\pi\nu_{ep}$ , where  $\omega_{pl} = \sqrt{4\pi e^2 n_e/m_e} = 5.65 \times 10^4 \sqrt{n_e} \text{ s}^{-1}$  is the plasma (Langmuir) frequency;  $\nu_{ep} = 50 n_e/T^{3/2} \text{ s}^{-1}$  is the electron-proton collision frequency, the electron density  $n_e (= n_p)$  (cm<sup>-3</sup>) and the temperature  $T$  (K).

Thus we find that the magnetic diffusion coefficient  $\nu_m \simeq 10^{13} (T/1 \text{ K})^{-3/2} \text{ cm}^2 \text{ s}^{-1}$  does not depend on the charge density  $n_e$  and it is very small relative to the product  $vl$  for hot plasma  $T \geq 10^5 \text{ K} \gg 1 \text{ K}$ . From the comparison of the first and second terms in the r.h.s. of the Faraday equation Eq. (3) we find that  $v/l \gg \nu_m/l^2$ , or  $\nu_m \ll vl \sim 10^{13} \text{ cm}^2 \text{ s}^{-1}$  since  $T/1 \text{ K} \gg 1$ . This means that the magnetic field in the Sun is mainly *frozen-in*. Neglecting the second term in Eq. (3) and using the Maxwell equation  $\text{rot} \vec{E} = -c^{-1}(\partial \vec{B}/\partial t)$  we obtain the condition for frozen-in field: the Lorentz force vanishes,  $\sim (\vec{E} + [\vec{v} \times \vec{B}]/c) \simeq 0$  but the current  $\vec{j} = \sigma_{cond}(\vec{E} + [\vec{v} \times \vec{B}]/c)$  remains finite if the conductivity is large,  $\sigma_{cond} \rightarrow \infty$ .

The magnetic Reynolds number

$$R_m = lv\omega_{pl}^2/(c^2\nu_{ep}) \simeq lv \times 10^{-13} (T/1 \text{ K})^{3/2} \text{ cm}^2 \text{ s}^{-1}$$

is huge if we substitute the estimate  $lv \sim 10^{13} \text{ cm}^2 \text{ s}^{-1}$  given above. A large value for the Reynolds number is a necessary condition for the existence of an effective dynamo enhancement in the convective zone.

The random magnetic field component in the Sun,  $\tilde{B}(t)$ , will be described in general by an arbitrary, phenomenological, correlator of the form  $\langle \tilde{B}(t)\tilde{B}(t') \rangle = \langle \tilde{B}^2 \rangle f(t - t')$ . We will assume that the strength of the r.m.s. field squared  $\langle \tilde{B}^2 \rangle = \eta B_0^2$  is parametrized by the dimensionless parameter  $\eta = R_m^\sigma > 1$ , which it can be, in general, much bigger than unity,  $\eta \gg 1$ . Actually, the estimation of the quantity  $\eta$ , the ratio of rms fields to regular field, for the solar convective zone (and other cosmic dynamos) is the matter of current scientific discussions. The most conservative estimate, simply based on equipartition, is  $\eta = \text{constant}$ . According to direct observations of galactic magnetic field presumably driven by a dynamo,  $\eta \simeq 1.8$  [13]. A more developed theory of equipartition gives  $\eta \simeq 4\pi \ln R_m$  (see Ref. [14]). Let us note that this estimate is considered now as very conservative: more detailed theories of MHD turbulence yield estimates like  $\eta \sim \sqrt{R_m}$  [8, 9].

The correlator function  $f(t)$  is unknown a priori but it takes the particular  $\delta$ -correlator form  $f(t) = L_0 \delta(t)$  if the correlation length (for two neighboring magnetic field domains) is much less than the neutrino oscillation length,  $L_0 \ll l_{osc}$ .  $L_0$  can be considered a free parameter ranging in the interval  $10^0 - 10^4$  km. In the averaged evolution equations it only appears the product  $\eta L_0$ . Thus in what follows we will present our results as a function of the quantity  $P$  which is a simple function of such a product:

$$P \equiv \frac{1}{2} (1 + \exp(-\gamma)), \quad \gamma \equiv \frac{4}{3} \Omega^2 \Delta t \equiv \frac{4}{3} \eta L_0 (\mu B_0)^2 \Delta t. \quad (4)$$

The reason for using the quantity  $P$  is that it is a good approximation for the depolarization that the presence of noise induces in the averaged neutrino density matrix.  $\Delta t$  is the distance over which the noise is acting. We have supposed in our computations that the noise is only effective in a thin layer with thickness  $\Delta t = 0.1 R_\odot$  starting at  $r = 0.7 R_\odot$ , the bottom of the convective zone. In Table (1) of Ref.([15]) the quantities  $\sqrt{\langle B_0^2 \rangle}$  and  $\eta$  are computed for a given  $P$  supposing the reasonable value  $L_0 = 1000$  Km. For example for  $P = 0.95$ ,  $\sqrt{\langle B_0^2 \rangle} = 10$  and  $\eta(\mu B = 1) = 100$  (all  $\mu B$  are given in  $\mu_{11} B_4$  units).

### 3.2 Regular large-scale magnetic field in the Sun. The twisting field

In this work, we will apply for the neutrino conversions described by our master equation Eq. (1) the self-consistent model of large-scale regular field given in Ref. [16]. In this model, the global solar magnetic field is the axisymmetric equilibrium solution of the MHD static equations (quiet Sun) in the spherically symmetric gravitational field of the Sun. The reasonable boundary condition  $B_0 = 0$  on the photosphere ( $r = R_\odot$ ) is imposed in addition. Any field solution to these equations and boundary conditions is a twisting field *with an, arbitrary, small or large number of revolutions along radius* ( $k = 1, 2, \dots$ , the twist rate can be taken as a label to distinguish particular solution within the family). Full expressions for the spherical components of the magnetic field can be found in Ref.([15]). There is only a free parameter in this model. The constant  $K$  is related with central field

$B_{core}$  by:  $K = B_{core}/2(1 - \alpha R_{\odot}/\sin \alpha R_{\odot})$ . The modulus of the perpendicular component is of the form:  $B_{0\perp} = B_{core} \sin \theta f(r)/r$  where  $f(r)$  is some known function of gentle behavior. According to this model, the expected magnetic field at the core is typically only 2-3 times (or less) the magnetic field at the convective zone. For the values that we will consider later,  $B_{0.7} \approx < 100 - 200$  kG, the values corresponding at the core are well below astrophysical bounds derived from traces of these fields at solar surface.

## 4 The averaged master equation

The master Equation (1) can be written in terms of the density matrix  $\rho(t)$  as:

$$i\partial_t \rho = [H_{reg}, \rho] + \mu \tilde{B}_x(t)[V_x, \rho] + \mu \tilde{B}_y(t)[V_y, \rho]. \quad (5)$$

The elements of the matrices  $H_0, V_x, V_y$  can be read off the Eq. (1). The  $\tilde{B}_x, \tilde{B}_y$  are the Cartesian transversal components of the chaotic magnetic field. Vacuum mixing terms and matter terms corresponding to the SSM density profile given before and the regular magnetic part Hamiltonian are all included in  $H_{reg}$ . In particular, the matrices  $V_{x,y}$  are Gamma-like matrices in terms of the Pauli matrices  $\sigma_{1,2}$  [15].

It is our objective in this section to write the differential evolution equation for the average density matrix  $\langle \rho \rangle$ . We assume that the components  $\tilde{B}_x, \tilde{B}_y$  are statistically independent, each of them characterized by a delta-correlation function ( $\langle \tilde{B}_x(t) \tilde{B}_y(t') \rangle = 0$ ).

$$\langle \tilde{B}_{x,y}(t) \tilde{B}_{x,y}(t') \rangle = \langle \tilde{B}_{x,y}^2 \rangle L_0 \delta(t - t'). \quad (6)$$

In addition, we will make an equipartition assumption for each of the three cartesian components  $\tilde{B}_{x,y,z}$ . The averaged evolution equation is a simple generalization (see Ref. [17] for a complete derivation) of the well known Redfield equation [18, 19, 20] for two independent sources of noise and reads ( $\Omega^2 \equiv L_0 \mu^2 \langle \tilde{B}^2 \rangle / 2 \equiv \eta L_0 (\mu B_0)^2 / 3$ ):

$$i\partial_t \langle \rho \rangle = [H_{reg}, \langle \rho \rangle] - i\Omega^2 [V_x, [V_x, \langle \rho \rangle]] - i\Omega^2 [V_y, [V_y, \langle \rho \rangle]]. \quad (7)$$

It is possible to write the Eq. (7) in a more evolved form. Taking into account the particular form of the matrices  $V_{x,y}$  and performing a rescaling of the density matrix given by:  $\langle \rho(t) \rangle = \exp(-4\Omega^2 t) \langle \rho'(t) \rangle$  we finally obtain the desired averaged evolution equation:

$$i\partial_t \langle \rho' \rangle = [H_{reg}, \langle \rho' \rangle] + i2\Omega^2 (V_x \langle \rho' \rangle V_x + V_y \langle \rho' \rangle V_y). \quad (8)$$

It is useful however to consider the solution to Eq. (8) when  $H_{reg} \equiv 0$ . This is the appropriate limit when dealing with extremely low  $\Delta m^2$  or very large energies, for an extreme level of noise or when the distance over which the noise is acting is small enough to consider the evolution given by  $H_{reg}$  negligible. In any other scenario it can give at least an idea of the general behavior of the solutions to the full Eq. (8). When  $H_0 = 0$  only the two last terms in the equation remain and an exact simple expression is obtainable by

ordinary algebraic methods. The full 4x4 Hamiltonian decouples in 2x2 blocks. If  $P_{f,i}$  are the final and initial probabilities (at the exit and at the entrance of the noise region) their averaged counterparts fulfill linear relations among them, schematically:  $Q_f^{A,B} = M Q_i^{A,B}$  with  $Q^{A,B}$  any of the two dimensional vectors  $Q^A = (\langle P(\nu_{eL} \rightarrow \nu_{eL}) \rangle, \langle P(\nu_{eL} \rightarrow \tilde{\nu}_{\mu R}) \rangle)$ ,  $Q^B = (\langle P(\nu_{eL} \rightarrow \tilde{\nu}_{eR}) \rangle, \langle P(\nu_{eL} \rightarrow \nu_{\mu L}) \rangle)$  and the Markovian matrix  $M$ :

$$M = \begin{pmatrix} P & 1-P \\ 1-P & P \end{pmatrix} \quad (9)$$

with  $P$  defined in Eq. (4). It can be shown that in this simple case  $P$  is exactly the final polarization of the density matrix (one of the eigenvalues of the matrix  $M$  is equal to  $2P - 1$ ). In the general case with a finite  $H_{reg}$  it can be shown numerically that the quantity  $P$  still gives a reasonable approximation ( $< 10\%$ ) to the real polarization, at least for the cases of interest in this work.

## 5 Results and Discussion

The present status of the Solar neutrino problem in Tables (1) of Refs. [15, 21]. For this data, we have calculated the expected neutrino signals in the Homestake, Ga-Ge and (Super)-Kamiokande experiments. For this objective, the time averaged survival and transition probabilities have been obtained by numerical integration of the ensemble averaged master equation (5) for a certain regular magnetic profile  $B_{0\perp}$  (see Ref.[15] for details). The free parameters of our model are four:  $\delta = \Delta m^2/2E$ ,  $s_2^2 \equiv \sin^2 2\theta$ , a noise strength parameter ( $P$ ) and the product of magnetic field and moment ( $\mu B_{0\perp}$  at  $r = 0.7R_\odot$ ,  $\theta_s = 7^\circ$ ).

In Figs. 1-2 we present the  $(\Delta m^2, \sin^2 2\theta)$  exclusion plots from a combined  $\chi^2$  analysis of the three experiments. First we comment the results in complete absence of noise ( $P=1$ ) which are represented in Fig. 1 (Left block of four plots). For negligible or low regular magnetic field we observe the high squared mass difference solutions proportioned by the matter MSW effect. As the magnitude of the magnetic field increases new solutions appear and disappear in a complicated manner. The low angle solution however disappears at high magnetic fields, after experiencing some distortion coming from its merging with newborn magnetic solutions (compare low angle allowed regions in Figures (b) and (c)). The antineutrino production (dashed lines) is in general low and Kamiokande bounds are not specially restrictive except at very high magnetic fields. Note in Figure (d) the very different behavior of the two existing allowed regions: while the in MSW region the antineutrino production is in the  $0.1 - 1\%$ , compatible comfortably with Kamiokande bounds, the RSFP solution reach a value well above  $10\%$  and is excluded by them. It seems apparent that there are acceptable particle solutions to the SNP even for very large regular magnetic fields.

The pattern of the electron antineutrino probability is very different when a small level of noise ( $P=0.95$ , Fig. 1, right block of four plots) is switched on. For  $\Delta m^2 > 10^{-6} \text{ eV}^2$ : the antineutrino iso-probability lines follow the characteristic MSW triangular patterns

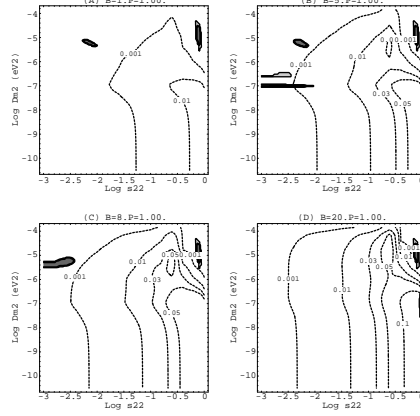


Figure 1: The parameter regions consistent with the Homestake, Kamiokande and combined Gallium experiments in absence of noise (Left Block of four figures):  $P = 1.00$ . (C.L.= 90%, 95%, 99% (from darkest to lighter shaded areas)). Figures (a,b,c,d) correspond to values of the regular magnetic field at the bottom of the convective zone:  $\mu B = 1, 5, 8, 20 \mu_{11} B_4$ . The electron antineutrino averaged probability, is represented by the dashed lines. Present Kamiokande bounds are  $P_{\nu_e \bar{\nu}_e} < 0.05$ . Right Block: the same for  $P = 0.95$ .

in this region. The structure of the allowed regions remain unmodified. The electron antineutrino yield in these regions is below the 5% level similarly as before. For stronger levels of noise ( $P=0.8$ , Figs. 2) the same comments can be said. The structure and position of the allowed regions from combined total rates are practically unmodified but the antineutrino yield impose strong restrictions. For an antineutrino probability smaller than 3% only some residual, 90% C.L., allowed regions exist at very small mixing angle,  $\Delta m^2 \approx 10^{-6} - 10^{-7} \text{ eV}^2$  and moderately high regular magnetic field [Figure (c)].

The same regions are still acceptable for  $P=0.7$ , (Figs. 2). For this level of noise something unexpected happens at extremely high regular magnetic field [Figure (d)] (probably too high to be acceptable on astrophysical grounds): a new, large, acceptable region appear for  $\Delta m^2 \approx 10^{-5}$ . This region disappears again for extremely high chaotic fields ( $P=0.55$ , Figure not shown here). Note that even for this value of  $P$  some residual regions with a 90% C.L. are marginally acceptable from reconciliation of all experiment total rates and antineutrino bounds [2] if the regular magnetic field is  $\approx 200 \text{ kG}$  (for  $\mu = 10^{-11} \mu_B$ ).

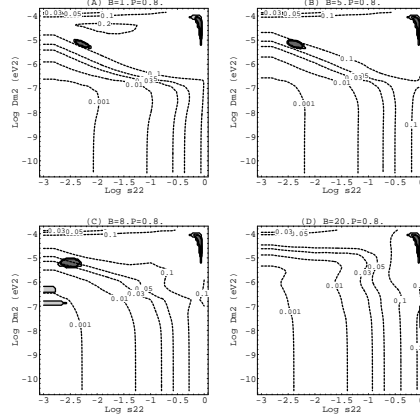


Figure 2: Left Block of four Figures: As Fig. 1 for  $P = 0.80$ . Right Block:  $P = 0.70$ .

## 6 Conclusions

We have presented calculations of neutrino spin flavor precession in presence a layer of magnetic noise at the bottom of the convective zone together with matter and a theoretically motivated solar magnetic field regular profile. We have justified that the level of noise in this region can be very high. We find that MSW regions ( $\Delta m^2 \approx 10^{-5} \text{ eV}^2$ , both small and large mixing solutions) are stable up to very large levels of noise ( $P=0.7-0.8$ ) but they are acceptable from the point of view of antineutrino production only for moderate levels of noise ( $P \approx 0.95$ ). The stronger r.m.s field occurs at the convective zone, the wider  $(\delta m^2, \sin^2 2\theta)$  region should be excluded when considering the constrain imposed by existing antineutrino bounds. For strong noise,  $P = 0.7$  or bigger and reasonable regular magnetic field, any parameter region  $(\Delta m^2, \sin^2 2\theta)$  is excluded. This model of noisy magnetic field is not compatible with particle physics solutions to the SNP. One is allowed then to reverse the problem and to put limits on r.m.s field strength, correlation length and transition magnetic moments by demanding a solution to the SNP under this scenario.

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